Deterministic and Probabilistic Analysis of the Foundation Plate and Soil Interaction

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Abstract: This paper deals with the influence of the input data uncertainties to the reliability of the foundation plate. The impact of the variability of the soil stiffness, structure geometry, permanent and variable masses are considered. The advantages and disadvantages of the deterministic and probabilistic analysis of the foundation plate resistance are discussed. The sensitivity of the foundation to the uncertainties of the soil properties due to longtime performance is not negligible for design engineers. On the example of building foundation plate the affectivity of the probabilistic design methodology was presented. The response surface method (RSM) for the analysis of the foundation plate reliability was used on program ANSYS. The probabilistic analysis gives us more complex information about the soil-foundation-structure interaction as the deterministic analysis.

Keywords: Probability, foundation plate, soil-structure interaction, RSM, FEM, ANSYS.

1. Introduction

The requirements to design of the plate foundation increased due to development of calculation method and computer tools. The calculation model and resistance uncertainties determine the optimal design of the foundation plate. During the structural design process, an engineer has to consider problems of the soil-foundation and foundation-structure interaction in the point of view safety, reliability and durability of the structures. In the case of the high rise buildings the design of the foundation has the significant effect to safety of buildings.

Randomness in the loading and the environmental effects, the variability of the material and geometric characteristics of structures and many other “uncertainties” affecting errors in the computing model lead to a situation where the actual behavior of a structure is different from the modeled one [1, 2, 7, 11, 14].

Recent advances and the general accessibility of information technologies and computing techniques give rise to assumptions concerning the wider use of the probabilistic assessment of the reliability of structures through the use of simulation methods [4, 6, 8, 9, 10]. Much attention should be paid to using the probabilistic approach in an analysis of the reliability of structures [3, 5, 12, 13].

Most problems concerning the reliability of building structures are defined today as a comparison of two stochastic values, loading effects $E$ and the resistance $R$, depending on the variable material and geometric characteristics of the structural element. The variability of those parameters is characterized by the corresponding functions of the probability density $f_E(x)$ and $f_R(x)$. In the case
of a deterministic approach to a design, the deterministic (nominal) attributes of those parameters $R_d$ and $E_d$ are compared.

The deterministic definition of the reliability condition has the form

$$R_d \geq E_d$$

(1)

and in the case of the probabilistic approach, it has the form

$$RF = R - E \geq 0$$

(2)

where $RF$ is the reliability function, which can be expressed generally as a function of the stochastic parameters $X_1, X_2$ to $X_n$, used in the calculation of $R$ and $E$.

$$RF = g(X_1, X_2, ..., X_n)$$

(3)

The failure function $g(X)$ represents the condition (reserve) of the reliability, which can either be an explicit or implicit function of the stochastic parameters and can be single (defined on one cross-section) or complex (defined on several cross-sections, e.g., on a complex finite element model).

The most general form of the probabilistic reliability condition is given as follows:

$$p_f = P(R - E < 0) \equiv P(RF < 0) < p_d$$

(4)

where $p_d$ is the so-called design (“allowed” or “acceptable”) value of the probability of failure. From the analytic formulation of the probability density by the functions $f_R(x)$ and $f_E(x)$ and the corresponding distribution functions $\Phi_R(x)$ and $\Phi_E(x)$, the probability of failure can be defined in the general form:

$$p_f = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int f_R(x) \Phi_R(x) dx = \int_{-\infty}^{\infty} \Phi_E(x) f_E(x) dx$$

(5)

This integral can be solved analytically only for simple cases; in a general case it should be solved using numerical integration methods after discretization.

Except of $p_f$, the target reliability index $\beta$ is used as the measure of reliability, which is defined on assumption of linear failure function $g(X)$. In the case of normal (or lognormal) histograms of this function, we have

$$\beta = \frac{\mu_{RF}}{\sigma_{RF}}$$

(6)

where $\mu_{RF}$ and $\sigma_{RF}$ are mean values and standard deviation of reliability function defined in the form
\[ \mu_{RF} = \mu_R + \mu_E \]
\[ \sigma_{RF}^2 = \sigma_R^2 + \sigma_E^2 \]  

(7)

The integral in the formulation (5) can be solved analytically only for simple cases (6); in a general case it should be solved using numerical integration methods after discretization. The reliability criteria for the design of structures are defined in the Eurocode in dependency on reliability index \( \beta \), what is adequate to target level of failure probability (Tab.1).

<table>
<thead>
<tr>
<th>Limit state</th>
<th>Target reliability index ( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 years</td>
</tr>
<tr>
<td>Ultimate</td>
<td>3.8 ( (p_f \approx 10^{-4}) )</td>
</tr>
<tr>
<td>Fatigue</td>
<td>1.5-3.8 ( (p_f \approx 10^{1 \div 10^{-4}}) )</td>
</tr>
<tr>
<td>Serviceability</td>
<td>1.5 ( (p_f = 10^{-1}) )</td>
</tr>
</tbody>
</table>

Tab.1. Target reliability index \( \beta \) and probability of failure by Eurocode 1990

In the case of the stochastic approach, various forms of analyses (statistical analysis, sensitivity analysis, probabilistic analysis) can be performed. Considering the probabilistic procedures, Eurocode 1 (Fig.1) recommends a 3-level reliability analysis.

\[
\mathbb{P}_f = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}_{\{g(X_i) \leq 0\}}
\]  

(8)

where \( N \) is the simulation number, \( g(.) \) is the failure function, \( \mathbb{I}_{\{.\}} \) is the function with value 1, if the condition in the square bracket is fulfilled, otherwise is equal to 0.

![Fig.1. Overview of reliability methods](image)
The variation of this failure estimation can be described by Melcher in the form

\[
\hat{x}^2_{pf} = \frac{1}{(N-1)} \left\{ \frac{1}{N} \sum_{i=1}^{N} I^2 \left[ g(X, \leq 0) \right] \right\} - \left\{ \frac{1}{N} \sum_{i=1}^{N} I \left[ g(X, \leq 0) \right] \right\}^2 \tag{9}
\]

2. Reliability Analysis Methods

From the point of view of one’s approach to the values considered, structural reliability analyses can be classified in two categories, i.e., deterministic analyses and stochastic analyses. In the case of the stochastic approach, various forms of analyses (statistical analysis, sensitivity analysis, probabilistic analysis) can be performed. Considering the probabilistic procedures, Eurocode 1 recommends a 3-level reliability analysis. The reliability assessment criteria according to the reliability index are defined here. Most of these methods are based on the integration of Monte Carlo (MC) simulations.

![Fig.2. Procedural diagram of probabilistic calculations using the ANSYS software system](image)
Three categories of methods have been presently realized:

- **Direct methods** (Importance Sampling - IS, Adaptive Sampling - AS, Direct Sampling - DS)
- **Modified methods** (Conditional, Latin Hypercube Sampling - LHS)
- **Approximation methods** (Response Surface Method - RSM)

The advantages and disadvantages of these methods are described in detail in the paper [9]. The ANSYS Program belongs among the complex programs for solving potential problems. It contains a postprocessor, which enables the execution of the probabilistic analysis of structures. In Fig. 1, the procedural diagram sequence is presented from the structure of the model through the calculations, up to an evaluation of the probability of structural failure.

The ANSYS postprocessor enables the modeling of a structure as a solid body having a general shape (solid modeling), using Boolean operations, general spline planes (non-uniform rational B-splines), automated meshing and adaptive meshing.

### 3. Calculation Model of the Foundation Plates

The resistance of the foundation plates of high rise buildings were investigated using the deterministic and probabilistic analyses. The building has 20 storey overground and 3 storey underground with storey height of 3m. The three types of the high rise building were considered. First model “D1” consists two cores and columns system, the foundation plate dimension is 21x36m. Second model “D2” consists two central cores and columns system, the foundation plate dimension is 21x30m. Third model “D3” consists the parallel walls a’ 6m, the foundation plate dimension is 21x36m.

All columns in these buildings are 600/600mm in cross-section. The thickness of floor reinforced concrete plate is 220mm. All floor slabs have a permanent load of 0,5kN/m² and variable load of 2,0kN/m². The material properties of this concrete building are Young’s modulus, \( E = 30 \text{GPa} \) and Poisson’s ratio \( \mu = 0,2 \).

The walls and foundation plate was modeled in software ANSYS using shell elements SHELL43, the Winkler subsoil by element SURFACE154 and the solid subsoil by element SOLID45.

![Fig. 3. Calculation models of foundation plate – D1, D2 and D3](image-url)
4. Soil-Foundation Interaction

The consideration of SSI effects is very important during the design process of the high rise building. The influence of the variability of the soil stiffness characteristic to the structure are not negligible. The subsoil was considered as the layered medium typical to the environs of the city Bratislava (tab.2). The stiffness of the original subsoil is poor for the foundation of the high rise buildings usually. Hence the technology of the soil upgrading is used. The system KELLER propose the effective technology of the soil upgrading (Tab.2).

<table>
<thead>
<tr>
<th>Point</th>
<th>Soil type</th>
<th>typ</th>
<th>$h_i$ [m]</th>
<th>$\gamma$ k[Nm$^{-3}$]</th>
<th>$\nu$</th>
<th>$c$ [kPa]</th>
<th>$\Phi_{ef}$ [deg]</th>
<th>Original subsoil</th>
<th>Strengthened subsoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>G2</td>
<td></td>
<td>2.7</td>
<td>15330</td>
<td>0.43</td>
<td>0</td>
<td>31</td>
<td>G2+</td>
<td>75735</td>
</tr>
<tr>
<td>2</td>
<td>CH</td>
<td></td>
<td>3.5</td>
<td>17810</td>
<td>0.42</td>
<td>10</td>
<td>16</td>
<td>CH+</td>
<td>33747</td>
</tr>
<tr>
<td>3</td>
<td>ML</td>
<td></td>
<td>1.0</td>
<td>12728</td>
<td>0.46</td>
<td>18</td>
<td>25</td>
<td>Soilcret</td>
<td>900000</td>
</tr>
<tr>
<td>4</td>
<td>ML</td>
<td></td>
<td>2.8</td>
<td>11142</td>
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<td>22</td>
<td>Soilcret</td>
<td>900000</td>
</tr>
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<td>10266</td>
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<td>10</td>
<td>28</td>
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<td>10266</td>
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</tr>
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<td>4.0</td>
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<td>10</td>
<td>28</td>
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<td>16</td>
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<td>SC</td>
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<td>0.35</td>
<td>10</td>
<td>28</td>
<td>SC</td>
<td>16200</td>
</tr>
</tbody>
</table>

Tab.2. The mechanical characteristic of the layered subsoil

The subsoil can be modeled by 3D FEM model or simple 1D Winkler model. In this paper the influence of the accuracy of these soil models to design of the foundation plate are considered. The most popular is the simple Winkler or Winkler-Pasternak model of the subsoil in practice. These models are based on the theory of the 1D soil columns. Winkler constant $C$ can be defined in the form

$$C = \int_a^b E_{oed} k \left( \frac{d\psi}{dz} \right)^2 dz = \frac{\sigma_c}{w}$$

(10)

where $E_{oed}$ is the characteristic oedometric modulus, $\psi$ is the shape function of the settlement along the depth of soil, $\sigma_c$ is the contact stress from the building under the foundation, $w$ is the settlement of the foundation. In the case of the layered soil the settlement may be calculated by STN 1001 using the following formula
where $\sigma_{z,i}$ is the vertical stress in the middle of soil layer under foundation, $m_i$ is the correct factor dependent on soil type, $\sigma_{or,i}$ is the original geostatic stress in the middle of soil layer, $h_i$ is the layer thickness, $E_{oed,i}$ is the oedometric modulus of $i^{th}$ soil layer.

The Winkler subsoil can be modeled using surface elements SURF154 in the system ANSYS. These elements can be defined around of plate area to consider the effect of the soil resistance. The size of the boundary area $L_w$ is determined as follows:

$$L_w = h_i \tan(\phi_{ef})$$

where $\phi_{ef}$ is the effective angle of the soil internal friction, $h_i$ is the active zone of subsoil under plate.

5. Loading and Load Combination

The loading and load combination in the case of the deterministic as well as the probability calculation is different due to requirements of Eurocode 1990 [R22] and JCSS 2000 [R44], too. In the case of deterministic calculation of the structure the load combination is considered according to ENV 1990 as follows:

**Fundamental combination – deterministic method**

$$E_x = \gamma_g G_k + \gamma_q Q_k + \gamma_s S_k$$

where $G_k$ is the characteristic value of the permanent loads, $Q_k$ - the characteristic value of the variable loading, $S_k$ - the characteristic value of the snow loading, $\psi_{os}$ - the combination factor according to ENV 1990 ($\psi_{os} = 0.6$). The load factors $\gamma_g$, $\gamma_q$, and $\gamma_s$ are considered for the ultimate limit state ($\gamma_g = 1.35$; $\gamma_q = 1.5$; $\gamma_s = 1.5$) and serviceability limit state ($\gamma_g = 1.0$; $\gamma_q = 1.0$; $\gamma_s = 1.0$) in accordance with requirements of ENV 1990.

In the case of *probabilistic calculation* of the structure the load combination we take following:

**Fundamental combination– probabilistic method**

$$E = G + Q + S = g_{var} G_k + q_{var} Q_k + s_{var} S_k$$

where $g_{var}$, $q_{var}$, $s_{var}$ are the variable parameters defined in the form of the histogram calibrated to the load combination in compliance with Eurocode [R22].

6. Uncertainties of Input Variables

The effect of soil-structure interaction can be investigated in the case of probabilistic assessment by sensitivity analysis of the influence of variable properties of soil. A soil stiffness variability in
the vertical direction is defined by the characteristic stiffness value \( k_{i,k} \) from the geological measurement and the variable factor \( k_{i,var} \). The stiffness of the structure is determined with the characteristic value of Young’s modulus \( E_k \) and variable factor \( e_{var} \). A load is taken with characteristic values \( G_k, Q_k, S_k \) and variable factors \( g_{var}, q_{var}, s_{var} \).

The uncertainties of the calculation model are considered by variable model factor \( \theta_R \) and variable load factor \( \theta_E \) for Gauss’s normal distribution. In view of the uncertainties of calculation soil model and mechanical soil properties the various stiffness of soil are requirement to design foundation plate. In the deterministic analysis are used three values of the soil stiffness (Low-Medium-High), which values are determined from the median and lower and upper two quantile of this stiffness. These values of the quantile \( k_p \) can be defined for the normal distribution as follow

\[
k_p = k_m \left( 1 \pm u_p \right)
\]

where \( k_p \) is the quantil of the soil stiffness (for probability \( p=0,05 \) and \( p=0,95 \)), \( k_m \) is the mean value of the soil stiffness, \( k_w \) is the value of the soil stiffness variation \( (k_w=k_o/k_m) \), \( u_p \) is the normalized of the quantil values. On the base of the requirements [] the standard deviation can be considered by 30% of mean value. The deterministic factors of the soil stiffness are utilized by values 0.5/1/1.5 (Low/Medium/High).

<table>
<thead>
<tr>
<th>Name</th>
<th>Quantity</th>
<th>Character value</th>
<th>Variable paramet.</th>
<th>Histogram</th>
<th>Mean</th>
<th>Stand. dev.</th>
<th>Min. value</th>
<th>Max. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>Layer stiffness</td>
<td>( k_{i,k} )</td>
<td>( k_{i,var} )</td>
<td>Normal</td>
<td>1.00</td>
<td>0.300</td>
<td>0.574</td>
<td>1.433</td>
</tr>
<tr>
<td>Material</td>
<td>Young’s modul.</td>
<td>( E_k )</td>
<td>( e_{var} )</td>
<td>Normal</td>
<td>1.00</td>
<td>0.100</td>
<td>0.526</td>
<td>1.407</td>
</tr>
<tr>
<td>Action</td>
<td>Permanent</td>
<td>( G_k )</td>
<td>( g_{var} )</td>
<td>Normal</td>
<td>0.60</td>
<td>0.350</td>
<td>0.005</td>
<td>4.073</td>
</tr>
<tr>
<td>Variable</td>
<td>( Q_{b1} )</td>
<td>( q_{var} )</td>
<td>Gama</td>
<td>0.35</td>
<td>0.245</td>
<td>0.003</td>
<td>1.953</td>
<td></td>
</tr>
<tr>
<td>Snow</td>
<td>( S_k )</td>
<td>( s_{var} )</td>
<td>Gama</td>
<td>0.35</td>
<td>0.245</td>
<td>0.003</td>
<td>1.953</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>Action uncertainty</td>
<td>( \theta_R )</td>
<td>( e_{var} )</td>
<td>Normal</td>
<td>1.00</td>
<td>0.100</td>
<td>0.526</td>
<td>1.407</td>
</tr>
<tr>
<td></td>
<td>Resist. uncertainty</td>
<td>( \theta_R )</td>
<td>( s_{var} )</td>
<td>Normal</td>
<td>1.00</td>
<td>0.100</td>
<td>0.526</td>
<td>1.407</td>
</tr>
</tbody>
</table>

Tab.3. Probabilistic model of input parameters

The random distribution of the input variables are considered on the base of the requirements ENV 1990 (tab.3). These values are calibrated to the ultimate limit state.
7. Reliability Criteria for Seismic Resistance of Structure

Reliability of the bearing structures is designed in accordance of standard requirements ENV 1992 for ultimate and serviceability limit state. The foundation reinforced concrete plate is designed on the bending and shear loads for ultimate limit state function as follow

\[ g(M) = 1 - \frac{M_E}{M_R} \geq 0 \]
\[ g(V) = 1 - \frac{V_E}{V_R} \geq 0 \]

where \( M_E, V_E \) are design bending moment and design shear force of the action and \( M_R, V_R \) are resistance bending moment and resistance shear force of the structure element.

The settlement \( w_E \) of the building is determined by the limit settlement \( w_R \) in accordance with ENV 1997 in the form

\[ g(w) = 1 - \frac{w_E}{w_R} \geq 0 \]

where \( w_E \) is the vertical displacement, \( w_R \) is the limit value of building settlement.

8. Sensitivity Analysis

Sensitivity analysis of the influence of the variable input parameters to the reliability of the structures depends on the statistical independency between input and output parameters. Matrix of correlation coefficients of the input and output parameters is defined by Spearman [8].

The results of the sensitivity analysis of the settlement and bending moment of action of the foundation plate to the fundamental load combination in the model D2c are presented in the Fig.4. These pictures present, that the variability of the input parameters \( q_{var}, g_{var} \) and \( s_{var} \) have the significant influence on the settlement and parameters \( q_{var}, g_{var}, e_{var} \), \( s_{var} \) and \( k_{4var} \) are important for the bending moment \( m_x \). A uncertainties of the variable load is first important parameter for the settlement and the bending moment and the variability of the permanent load for the shear forces.

Fig.4. Sensitivity analysis of the foundation plate settlement \( w \) and bending moment \( m_x \)
The ratio of the variability of input parameters is different in the model of foundation plate D1, D2 and D3. The configuration of the building walls and columns has the impact to the behavior of the internal forces of the foundation plate. In the case of the shear and bending resistance sensitivity analysis of the plate, the most important variable parameters are the resistance $r_{var}$ and $q_{var}$, $g_{var}$, $k_{4, var}$ also. The sensitivity analysis gives the valuable information about the influence of uncertainties of input variables (load, material, model,) to engineer for optimal design of the structures.

9. Comparison of Deterministic and Probabilistic Analyses

Deterministic calculation of the resistance of the foundation plate for three type of wall and column configuration was realized on the models D1, D2 and D3. The calculation model of the subsoil was considered as a) Winkler under plate area, b) Winkler under plate area and around strip, c) Solid under and around of plate area. The stiffness of Winkler constants are calculated from the equation (10). In the case of the soil model (b) the full and reduced stiffness (br) were considered. The reduction is defined by the ratio of the plate area and total subsoil area

$$k_{red} = \frac{A_p}{A_p + A_s}$$

where $A_p$ is plate area, $A_s$ is the subsoil area around of foundation plate.

<table>
<thead>
<tr>
<th>Model</th>
<th>$w_{max}$</th>
<th>$M_{max}$</th>
<th>$V_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[mm]</td>
<td>[%]</td>
<td>[kNm]</td>
</tr>
<tr>
<td>D1</td>
<td>a</td>
<td>-136,09</td>
<td>118</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-116,89</td>
<td>101</td>
</tr>
<tr>
<td></td>
<td>br</td>
<td>-161,39</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>-115,48</td>
<td>100</td>
</tr>
<tr>
<td>D2</td>
<td>a</td>
<td>-90,83</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-89,04</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>br</td>
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<tr>
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<td>c</td>
<td>-169,23</td>
<td>100</td>
</tr>
<tr>
<td>D3</td>
<td>a</td>
<td>-73,15</td>
<td>71</td>
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<td></td>
<td>b</td>
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<td>37</td>
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<td></td>
<td>br</td>
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<td>58</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>-102,85</td>
<td>100</td>
</tr>
</tbody>
</table>

Note: Winkler model=a),b),br); Solid model=c)

Tab.4. Comparison of deterministic analyses of the foundation plate
The results from the deterministic analysis with the different soil model (see Tab.4) show us that the maximal difference is equal 63% of the settlement in the model D3, 68% of the bending moment in the model D2, 38% of the shear forces in model D2. The scatter of the values of the output quantities is dependent on the structural system too.

The comparison of the deterministic and probabilistic analyses on the same soil model D3c show us that the maximal difference between the 95% quantil and mean deterministic value of the output quantity is equal 83% of the settlement in the model D3, 7% of the bending moment in the model D2, 13% of the shear forces in model D1. The scatter of the values of the internal forces from the deterministic and probabilistic analysis on the same soil model is lower as the difference between various soil models. The probabilistic results give us the less conservative values than the deterministic analysis.

### 10. Conclusions

This paper deals with the possibility of the probabilistic and sensitivity analysis of the reliability of the foundation plate depending on variability of the soil stiffness, structure geometry and action impact. The sensitivity of the foundation to the uncertainties of the soil properties is not negligible for design engineers. On the example of three type of the high rise buildings the affectivity of the probabilistic design methodology was presented. The approximation method RSM of simulation for the analysis of the foundation plate reliability was used on program ANSYS. The scatter of the output quantities between Winkler simple model and solid soil model is higher than the differences between the deterministic and probabilistic analysis in the same soil model. The probabilistic analysis gives us more complex information about the soil-structure interaction than the deterministic analysis.

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11. References


